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APPROXIMATION IN LINEAR DIFFERENCE- DIFFERENTIAL GAMES

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Methods are cited for the approximate solution of position difference - differential encounter games and for the exact solution of evasion games [1, 2], based on the use of certain finite-dimensional procedures for the determination of the control [3, 4]. The paper adjoins the investigations in [1—11].

1. We examine the controlled system

$$x^{\cdot}(t) = Ax(t) + A_\tau x(t-\tau) + B(t)u - C(t)v + w(t) \tag{1.1}$$
$$t_0 \leqslant t \leqslant \vartheta, \quad u \in P \subset E_{r_1}, \quad v \in Q \subset E_{r_2}$$
$$\tau = \text{const} > 0$$

Here x is the n-dimensional phase vector; the vectors u and v are the first and second players' controls, respectively; P and Q are convex compacta; A and A_τ are constant matrices and $B(t)$ and $C(t)$ are continuous matrices; $w(t)$ is a given perturbation (a Lebesgue - integrable function). The initial state $x_0(s) \in H$ [1, 2] and a closed set $M \subset E_n$ are prescribed. By choosing control u the first player strives to ensure the inclusion $x(t_*) \in M$ for at least one $t_* \in [t_0, \vartheta]$ (the game of encounter by an instant) or the inclusion $x(\vartheta) \in M$ (the game of encounter at an instant) for the system's phase vector. The second player strives to choose his control v so that the inclusion $x(t) \in M$ is not satisfied for any $t \in [t_0, \vartheta]$ (the evasion game).

The position p, the strategies U and V and the motions $x[t; p_0, U]$ and $x[t; p_0, V]$, $p_0 = \{t_0, x_0(s)\}$ have been defined in [2, 5]. The formalized statements of the problems (1) of encounter with set M at instant ϑ, (2) of encounter with set M by instant ϑ and (3) of evasion of set M, have been presented in the same References. From the results in these papers it follows that if strongly u-stable (u-stable) sets $K_t \subset H$, $t_0 \leqslant t \leqslant \vartheta$, $K_{\vartheta 0} \subset M$ and $x_0(s) \in K_{t_0}$ are prescribed, then the strategy U_e extremal to them solves problem 1 (problem 2). Here $K_{\vartheta 0}$ is the 0 section [6] of set K_ϑ. A similar result holds for problem 3 [10]. The determination of the control $u(t, x(s))$ on the strength of strategy U_e requires us to solve a certain extremal problem in Hilbert space H. We indicate below methods for constructing the first and second player's strategies ensuring the exact solution of problem 3 and the approximate solution (to any degree of accuracy) of problems 1 and 2; these methods are based on the solving of certain finite-dimensional extremal problems.

Let $X(t_0, x_0(s))$ be the sheaf of all motions $x(t) = x(t; p_0, u, v)$, $u \in \{u(\cdot)\}$ and $v \in \{v(\cdot)\}$ [1]; $X(t_*) = \{y(s) = x(t_* + s) \mid x(t) \in X(t_0, x_0(s))\}$ be the section of sheaf $X(t_0, x_0(s))$ by the hyperplane $t = t_*$; $T_m : H \to E_{(m+1)n}$ be the following operator:

$$T_m x(s) = \left\| \begin{matrix} y(0) \\ \vdots \\ y^{(m)} \end{matrix} \right\|, \quad y^{(0)} = x(0), \quad \omega_m = \frac{\tau}{m}$$

$$y^{(i)} = \omega_m^{-1/2} \int_{-i\omega_m}^{-(i-1)\omega_m} x(s)\,ds, \quad i = 1, \ldots, m$$

$$F_m(x(s)) = \sum_{i=1}^{m} \| y^{(i)} \|^2$$

$$\| x(s) \|_{m,\tau} = (F_m(x(s)) + \| y^{(0)} \|^2)^{1/2}$$

$$L_t = K_t \cap X(t) \neq \varnothing$$

D^ε be the closed ε-neighborhood of set D; $\| x(s) \|_\tau$ be the norm in H [1,2]; and K_t, $t_0 \leqslant t \leqslant \vartheta$, be a system of closed sets in H. Because the sheaf $X(t_0, x_0(s))$ is compact in $C^n[t_0, \vartheta]$ [1], from a number $\varepsilon > 0$ we can find a number $\beta = \beta(\varepsilon, x_0(s)) > 0$ such that the inequality

$$\| x(t_1) - x(t_2) \| \leqslant {}^1\!/_2\varepsilon \tag{1.2}$$

holds for any motion $x(t) \in X(t_0, x_0(s))$ and any instants t_1 and t_2, $| t_1 - t_2 | \leqslant \beta$. Using (1.2), by direct bounds we verify

Lemma 1.1. *The inequality*

$$\left| \int_{-\tau}^{0} \| x_1(t+s) - x_2(t+s) \|^2 ds - F_m(x_1(t+s) - x_2(t+s)) \right| \leqslant 2\tau\varepsilon^2$$

is valid for any $m \geqslant \tau / \beta$ *and* $x_i(t) \in X(t_0, x_0(s))$, $i = 1, 2$ *and* $t \in [t_0, \vartheta]$.

If the sets K_t, $t_0 \leqslant t \leqslant \vartheta$, are convex and closed, then for any element $x(s)$ we can find the unique elements $y_m(s; t, x(s))$ and $y(s; t, x(s))$ with properties

$$\| x(s) - y(s; t, x(s)) \|_\tau = \min_{y(s) \in L_t} \| x(s) - y(s) \|_\tau$$

$$\| x(s) - y_m(s; t, x(s)) \|_{m,\tau} = \min_{y(s) \in L_t} \| x(s) - y(s) \|_{m,\tau}$$

Taking Lemma 1.1 and Theorem 1.2 of [12] into account, for all $t \in [t_0, \vartheta]$, $m \geqslant \tau / \beta$ and $x(s) \in X(t)$ we obtain the estimate

$$\| y(s; t, x(s)) - y_m(s; t, x(s)) \|_\tau \leqslant \varkappa \tag{1.3}$$

$$\varkappa = (4\varepsilon\sqrt{\tau}B_1 + 4\varepsilon^2\tau)^{1/2}, \quad B_1 = \sup\{\| x_1(s) - x_2(s) \|_\tau \mid x_i(s) \in X(t),\ t \in [t_0, \vartheta],\ i = 1, 2\}$$

uniform relative to any system of closed sets $K_t \subset H$, $t_0 \leqslant t \leqslant \vartheta$, such that $L_t \neq \varnothing$.

We define strategy U_m as follows:

$$U_m(t, x(s)) = \{u_m \in P \mid B(t) u_m (z^{(0)} - y^{(0)}) = \max_{u \in P} B(t) u (z^{(0)} - y^{(0)})\}, \quad y = T_m x(s)$$

Here z is the vector from set $T_m L_t$ closest to vector y.

Theorem 1.1. Let the convex and closed sets K_t, $t_0 \leqslant t \leqslant \vartheta$, be strongly u-stable (be u-stable), $K_{\vartheta 0} \subset M$ and $x_0(s) \in K_{t_0}$. Then, for any number $\varepsilon > 0$ we can find a number $m_0 = m_0(\varepsilon, x_0(s))$ such that strategy U_m ensures that all motions $x[t; p_0, U_m]$ fall into the ε-neighborhood of set M at instant ϑ (by instant ϑ) for any $m > m_0$.

The theorem's proof follows the plan of the proof of Lemma 2.1 in [6], with the use of the obvious equality $z = T_m y_m(s; t, x(s))$ and of estimate (1.3). We note that estimate (1.3) is true if we consider

$$B_1 = \sup\{\min_{y(s)\in L_t} \|x(s) - y(s)\|_\tau \mid x(s) \in X(t), t \in [t_0, \vartheta]\}$$

It is easy to see that the above results remain true for nonlinear systems with aftereffect satisfying the conditions in [11].

2. Let us show that to solve problem 1 (problem 2) approximately from any position from which it is solvable we can use the strategy of aiming at certain sets constructed on the basis of position absorption sets for certain approximating systems without time lag. Together with system (1.1) we consider the following approximating systems:

$$dy^{(0)}/dt = Ay^{(0)} + A_\tau \omega^{-1/2} y^{(m)} + B(t)u - C(t)v + w(t) \qquad (2.1)$$

$$dy^{(1)}/dt = \omega_m^{-1/2} y^{(0)} - \omega_m^{-1} y^{(1)}$$

$$dy^{(i)}/dt = \omega_m^{-1} y^{(i-1)} - \omega_m^{-1} y^{(i)}, \quad i = 2, \ldots, m$$

$$t_0 \leqslant t \leqslant \vartheta, \quad u \in P, \quad v \in Q$$

Let $W_t^* \subset H$ and $W_t^*(\varepsilon) \subset H$ be sets of position absorption at instant ϑ (by instant ϑ) of sets M and M^ε, respectively, by system (1.1) [1,5] and $W_{mt}(\varepsilon)$ be the sets of position absorption at instant ϑ (by instant ϑ) by system (2.1) of the following set [3,4]:

$$M_\varepsilon^* = \{y = (y^{(0)}, \ldots, y^{(m)}) \in E_{(m+1)n} \mid y^{(0)} \in M^\varepsilon, \quad y^{(i)} \in E_n, \quad i = 1, \ldots, m\}$$

Taking the estimates in [7,8] into account, we can verify the validity of the following statement.

Lemma 2.1. For any $\alpha > 0$ we can find a number $N = N(\alpha)$ such that for any number $m > N$ the inclusions

$$W_t^* \cap X(t) \subset [T_m^{-1} W_{mt}(\alpha)] \cap X(t) \subset W_t^*(2\alpha) \cap X(t)$$

are fulfilled for all $t \in [t_0, \vartheta]$.

There holds

Lemma 2.2. For any $\varepsilon > 0$ and $t \in [t_0, \vartheta]$ there exists $\delta = \delta(t, \varepsilon) > 0$ ensuring the fulfillment of the relation

$$W_t^*(\delta) \cap X(t) \subset [W_t^* \cap X(t)]^\varepsilon$$

We denote $L_{mt}(\alpha) = [T_m^{-1} W_{mt}(\alpha)] \cap X(t)$, $y(s; t, x(s), m, \alpha)$ and $w(s; t, x(s), m, \alpha)$ are elements with the properties

$$\| x(s) - y(s; t, x(s), m, \alpha) \|_\tau = \min_{y(s) \in L_{mt}(\alpha)} \| x(s) - y(s) \|_\tau$$

$$\| x(s) - w(s; t, x(s), m, \alpha) \|_{m,\tau} = \min_{y(s) \in L_{mt}(\alpha)} \| x(s) - y(s) \|_{m,\tau}$$

We assume that sets $W_{mt}(\alpha)$ are convex. Then, as follows from (1.4), for any $\varkappa > 0$ we can find a number $N = N(\varkappa)$ such that the estimate

$$\| y(s; t, x(s), m, \alpha) - w(s; t, x(s), m, \alpha) \|_\tau < \varkappa \tag{2.2}$$

holds for all $m > N$, $t \in [t_0, \vartheta]$, $\alpha > 0$, $x(s) \in L_{mt}(\alpha)$ and $x(s) \in X(t)$. On the basis of Lemmas 2.1 and 2.2 and of Theorem 1.2 of [12], with due regard to the boundedness of $c_1 = \sup \{\| x_1(s) - x_2(s) \|_\tau \mid, \ x_i(s) \in X^\varepsilon(t), \ i = 1, 2; t \in [t_0, \vartheta]\}$, we get that for any $\varepsilon > 0$ and $t \in [t_0, \vartheta]$ we can find a number $\delta = \delta(t, \varepsilon) > 0$ such that for each $\alpha \in (0, \delta)$ we can find a number $N = N(\alpha)$ satisfying the condition: for every $m > N$ the inequality

$$\| y_*(s; t, x(s)) - y(s; t, x(s), m, \alpha) \|_\tau < \varepsilon \tag{2.3}$$

is valid for any element $x(s) \in X(t)$. Here $y_*(s; t, x(s))$ is the element of $W_t^* \cap X(t)$, closest to $x(s)$ in H.

We define the strategy U^ε as follows:

$$U^\varepsilon(t, x(s)) = \{u_* \in P \mid B(t) u_* (z^{(0)} - y^{(0)}) = \max_{u \in P} B(t) u (z^{(0)} - y^{(0)})\}, \quad y = T_{m_*} x(s)$$

Here z is the vector from $W_{m_* t}(\alpha_*) \cap T_{m_*} X(t)$, closest to vector y, α_* is some number from the interval $(0, \delta(t, \varepsilon))$ and m_* is some number greater than $N(\alpha_*)$.

Theorem 2.1. Let $x_0(s) \in W_{t_0}^*$. Then for any $\sigma > 0$ we can find $\varepsilon > 0$ such that the strategy U^ε ensures that the motions $x[t] = x[t; p_0, U^\varepsilon]$ fall into M^σ at instant ϑ (by instant ϑ).

The proof of this theorem is based on estimates (2.2) and (2.3) and on the equality $z = T_{m_*} w(s; t, x(s), m_*, \alpha)$ and follows the scheme of the proof of Theorem 2.1 in [6].

We define the strategy $U_{m,\alpha}$ as follows:

$$U_{m,\alpha}(t, x(s)) = \{u_* \in P \mid B(t) u_* (z_*^{(0)} - y_*^{(0)}) = \max_{u \in P} B(t) u (z_*^{(0)} - y_*^{(0)})\}, \quad y_* = T_m X(t)$$

Here z_* is the vector from $W_{mt}(\alpha) \cap T_m X(t)$, closest to y_*. From Theorem 2.1 follows

Theorem 2.2. Let $x_0(s) \in W_{t_0}^*$. Then for any $\varepsilon > 0$ we can find a number $\delta > 0$ with the property: for any finite partitioning Δ of the segment $[t_0, \vartheta]$, with diameter $\delta(\Delta) \leqslant \delta$, we can find $\alpha > 0$ and $N = N(\varepsilon, \Delta) > 0$ such that the strategy $U_{m,\alpha}$ ensures that all the approximating motions $x_\Delta[t] = x_\Delta[t; p_0, U_{m,\alpha}]$ [5,6] fall into M^ε at instant ϑ (by instant ϑ) if only $m > N$.

3. Let us establish that problem 1 for system (1.1) is equivalent to the same problem for a certain linear system without time lag of the same dimensionality. In this and subsequent sections the matrices $A = A(t)$ and $A_\tau = A_\tau(t)$ are continuous in t. Let $F(t, \xi)$ be the fundamental matrix of system (1.1) [1]; $A_{t_*, t^*}: H \to H$ be the solution operator of the homogeneous system corresponding to (1.1) [9]; $D_{t_*, t^*}: H \to E_n$ be the following operator $D_{t_*, t^*} x(s) = y(0)$, where $y(s) = A_{t_*, t^*} x(s)$. It is easy to prove the next statement by using the properties of matrix $F(t, \xi)$ [1].

Lemma 3.1. The equality

$$A_{t^*, \zeta} \int_{t_*}^{t^*} F(t^* + s, \xi) z(\xi) d\xi = \int_{t_*}^{t^*} F(\zeta + s; \xi) z(\xi) d\xi$$

holds for any summable n-dimensional function $z(t)$ and any $t_* \leqslant t^* \leqslant \zeta$ from $[t_0, \vartheta]$

Together with system (1.1) we consider the system without time lag

$$y^{\cdot} = F(\vartheta, t)(B(t)u - C(t)v + w(t)) \qquad (3.1)$$
$$t_0 \leqslant t \leqslant \vartheta, \quad u \in P, \quad v \in Q$$

The following strategy U^*, constructed for system (1.1),

$$U^*(t, x(s)) = \{u^* \in P \mid F(\vartheta, t) B(t) u^* (z - D_{t,\vartheta} x(s)) = \max_{u \in P} F(\vartheta, t) B(t) u (z - D_{t,\vartheta} x(s))\}$$

is called the strategy extremal to the closed sets $Z_t \subset E_n$, $t_0 \leqslant t \leqslant \vartheta$. Here z is the vector from Z_t, closest to the vector $D_{t,\vartheta} x(s)$. The strategy V^* extremal to sets Z_t is defined similarly. Using Lemma 3.1 we can prove

Lemma 3.2. Let the closed sets $Z_t \subset E_n, t_0 \leqslant t \leqslant \vartheta$, be strongly u-stable (strongly v-stable) for system (3.1) and let $D_{t_0,\vartheta} x_0(s) \in Z_{t_0}$. Then the strategy U^* (V^*) extremal to sets Z_t ensures that all motions $x[t] = x[t; p_0, U^*]$ ($x[t] = x[t; p_0, V^*]$) of system (1.1) hit onto sets Z_ϑ at instant ϑ.

Let W_t^* and W_t be the sets of position absorption of M by systems (1.1) and (3.1), respectively, at instant ϑ.

Theorem 3.1. $x_0(s) \in W_{t_0}^*$ if and only if

$$D_{t_0,\vartheta} x_0(s) \in W_{t_0} \qquad (3.2)$$

If (3.2) is fulfilled, the strategy U^* extremal to W_t solves problem 1. The proof of the theorem is based on Theorem 17.1 on the alternative in [3] and on Lemma 3.2.

4. Let us indicate another method for solving problem 2 approximately. Let $t_0 = \xi_0 < \ldots < \xi_m = \vartheta$, $\xi_{i+1} - \xi_i = \omega_m^* = (\vartheta - t_0)/m$, $i = 0, \ldots, m-1$. We consider the system without time lag

$$y_i^{\cdot} = \begin{cases} F(\xi_i, t)[B(t)u - C(t)v + w(t)], & t \in [t_0, \xi_i] \\ 0, & t \in (\xi_i, \vartheta] \end{cases} \qquad (4.1)$$

$$i = 1, \ldots, m \quad t_0 \leqslant t \leqslant \vartheta, \quad u \in P, \quad v \in Q$$

Let $i(t) = \min\{i \mid \xi_i \geqslant t, \ i = 1, \ldots, m\}$; $D_t^{(m)}: H \to E_{(m-i(t)+1)n}$ be the following operator:

$$D_t^{(m)} x(s) = \begin{Vmatrix} D_{t, \xi_{i(t)}} x(s) \\ \vdots \\ D_{t, \xi_m} x(s) \end{Vmatrix}$$

$$L^{(m)}(t) = \begin{Vmatrix} F(\xi_{i(t)}, t) \\ \vdots \\ F(\xi_m, t) \end{Vmatrix}$$

The strategy U_* constructed for system (1.1) by the rule

$$U_*(t, x(s)) = \{u_* \in P \mid L^{(m)}(t)B(t)u_*(z - D_t^{(m)} x(s)) = \max_{u \in P} L^{(m)}(t)B(t)u(z - D_t^{(m)} x(s))\}$$

is called the strategy extremal to the closed sets $Z_t \subset E_{mn}$. Here z is vector from $\pi_t^{(m)} Z_t$, closest to $D_t^{(m)} x(s)$, where $\pi_t^{(m)}: E_{mn} \to E_{(m-i(t)+1)n}$ is the operator of projection onto the last $(m - i(t) + 1)n$ coordinates. The strategy V^* extremal to Z_t is defined analogously.

For an arbitrary $\varepsilon > 0$ we choose the number $\delta(\varepsilon) > 0$ so as to fulfil the bound

$$\| D_{t_*, t^*} x(s) - x(0) \| < \varepsilon, \quad | t^* - t_* | < \delta(\varepsilon), \quad x(s) \in X(t_*) \tag{4.2}$$

The number $\delta(\varepsilon)$ exists by virtue of the compactness of sheaf $X(t_0, x_0(s))$ in $C^n[t_0, \vartheta]$. We introduce the sets

$$N_i^{(m)}(\alpha) = [\xi_i, \xi_{i+1}] \times E_{(i-1)n} \times M^\alpha \times E_{(m-i)n}, \quad i = 1, \ldots, m$$

$$N^{(m)}(\alpha) = \bigcup_{i=1}^{m} N_i^{(m)}(\alpha)$$

assuming $\alpha > 0$. Obviously, set $N^{(m)}(\alpha)$ is closed in $[t_0, \vartheta] \times E_{mn}$. We denote: $p_t^{(m)}: E_{mn} \to E_n$ is the operator of projection onto the coordinates numbered $(i(t) - 1)n + 1, \ldots, i(t)n$. Similarly to Lemma 3.2, using estimate (4.2) we can establish

Lemma 4.1. Let $\omega_m^* < \delta(\varepsilon)$, the closed sets $Z_t \subset E_{mn}$ be u-stable relative to $N^{(m)}(\alpha)$ (be strongly u-stable) for system (4.1) and $D_{t_0}^{(m)} x_0(s) \in Z_{t_0}$. Then the strategy U_* (V_*) extremal to sets Z_t ensures that the following condition is fulfilled for the motions $x[t] \equiv x[t; p_0, U_*]$ ($x[t] = x[t; p_0, V_*]$) of system (1.1): $x[t_*] \in M^{\alpha+\varepsilon}$ for some $t_* \in [t_0, \vartheta]$ ($x[t_*] \in (p_{t_*}^{(m)} Z_{t_*})^\varepsilon$ for all $t_* \in [t_0, \vartheta]$).

We introduce notation: W_t^* are sets of position absorption of M by system (1.1) by instant ϑ; $W_t^{(m)}(\alpha)$ are sets of position of absorption of M by system (4.1) by instant ϑ.

Theorem 4.1. If $x_0(s) \in W_{t_0}^*$, then for any $\varepsilon > 0$, $\alpha > 0$ and m such that $\omega_m^* < \min\{\delta(\alpha / 2), \delta(\varepsilon)\}$ the strategy U_* extremal to $W_t^{(m)}(\alpha)$ ensures that the motions $x[t] = x[t; p_0, U_*]$ fall into $M^{\alpha+\varepsilon}$ by instant ϑ. The theorem can be established by using Lemma 4.1 and the properties of sets $W_t^{(m)}(\alpha)$.

5. Let us study the problem 3 on evasion of target M, assuming the existence of a set R satisfying the relation $P = Q + R$. Let $Y(t, \xi)$ be the fundamental matrix of the homogeneous system corresponding to system (2.1); $Y_k^0(t, \xi)$ be the matrix composed from the first k rows of matrix $Y(t, \xi)$; $A^*_{m_t} : H \to E_{(m_t+1)n}$ be an operator of the form

$$A^*_{m_t} x(s) = \{y^{(0)}, \ldots, y^{(m_t)}\}, \quad y^{(0)} = x(0)$$

$$y^{(i)} = \omega_m^{-1/2} \int\limits_{-i\omega_m}^{-(i-1)\omega_m} x(s)\, ds, \quad i = 1, \ldots, m_t$$

$$y_0^* = A^*_{m_\theta} x_0(s)$$

$$K_l^\circ(t) = \left\{\int\limits_{t_0}^{t} Y_l^\circ(t, \xi)\, g^*(\xi)\, d\xi \mid g^*(\xi) \in R_*\right\} \subset H$$

$$L_l^\circ(t) = Y_l^\circ(t, t_0)\, y_0^* + K_l^\circ(t) + \int\limits_{t_0}^{t} Y_l^\circ(t, \xi)\, w_*(\xi)\, d\xi$$

$$m_t = \max\{i \mid t - i\omega_m \geqslant t_0,\ i = 0, \ldots, m\}$$

$$w_*(t) = (w(t), 0, \ldots, 0) \in E_{(m+1)n}$$

$$R_* = R \times \prod_{i=1}^{m} \{0\}$$

Here $\{0\}$ is the set consisting of the null vector of space E_n.

We define the strategy V_m by the system of sets $V_m(t, x(s))$ of the form

$$V_m(t, x(s)) = \{v^* \mid (g^{(0)}(t, x(s)) - [A^*_{m_t} x(s)]^{(0)})\, v^* = \max_{v \in Q} (g^{(0)}(t, x(s)) - [A^*_{m_t} x(s)]^{(0)})\, v\}$$

Here $g(t, x(s)) = \{g^{(0)}(t, x(s)), \ldots, g^{(m_t)}(t, x(s))\}$ is the element of set $A^*_{m_t} L_n^\circ(t + s)$, closest to $A^*_{m_t} x(s)$.

Theorem 5.1. For some number $\varepsilon > 0$ let $L_n^\circ(t) \cap M^\varepsilon = \varnothing$ for any instant $t \in [t_0, \vartheta]$. Then we can find a number N such that the strategy V_m solves problem 3 for all $m > N$.

The theorem's proof relies on the results in [7, 10]. We note that when set M is convex the hypothesis on Theorem 5.1 is fulfilled if the quantity $\varepsilon(t_0, t, x_0(s))$ defined in [2] is greater than zero for all $t \in [t_0, \vartheta]$.

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